DEPARTMENT OF PHYSICS
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PH3500 Classical Physics **Problem Set 8** **26.10.2009**

1. An atom with angular momentum quantum number j has $(2j+1)$ equally-spaced energy levels in an applied magnetic field. If B denotes the magnitude of the field, these levels are given by $\varepsilon_r = g\mu_B Br$ where $r = -j, -j+1, \ldots, (j-1), j$; g is a constant called the g-factor of the atom; μ_B is the Bohr magneton. Consider a paramagnetic material consisting of N such atoms, mutually non-interacting.

   (a) Show that the partition of the system is
   $$Z = z^N \quad \text{where} \quad z = \Big[\sinh(2j+1)\xi/\sinh\xi\Big], \quad \xi = \beta g\mu_B B/2 \ .$$

   (b) Hence show that the magnetization $M = -\Big(\partial F/\partial B\Big)_T$ is given by
   $$M = Ng\mu_B\left[\frac{1}{2}(2j+1)\coth(2j+1)\xi - \frac{1}{2}\coth\xi\right] \ .$$
   The quantity in the square brackets is sometimes called the Brillouin function $\mathcal{B}_j$.

   (c) The isothermal paramagnetic susceptibility $\chi_T = \Big[(\partial M/\partial B)_T\Big]_{B=0}$ can then be calculated. Show that the answer is
   $$\chi_T = Ng^2\mu_B^2 j(j+1)/(3k_B T) \ .$$
   The effective number of Bohr magnetons of each atom (or ion) of such a paramagnetic substance is thus $g[j(j+1)]^{1/2}$. Careful experiments on rare earth salts show good agreement between the calculated and experimental values. Typical numerical values for these rare earth ions range between 1 and 10.

2. At the critical point on the liquid-vapour co-existence curve, we have $(\partial P/\partial V)_T = 0$ (so that κ_T, the compressibility, becomes infinite). We also have $(\partial^2 P/\partial V^2)_T = 0$ (an inflection point on the isotherm). Does this imply that $(\partial^2 V/\partial P^2)_T$ is infinite at the critical point? Give a general answer; check out what happens in the case of the van der Waal's equation(given below).

3. Find the critical parameters T_c, V_c and P_c for the van der Waal's equation of state
   $$\left(P + \frac{aN^2}{V^2}\right)(V - Nb) = Nk_B T \ .$$
   Define reduced variables $t \equiv \frac{T-T_c}{T_c}$, $v \equiv \frac{V-V_c}{V_c}$ and $p \equiv \frac{P-P_c}{P_c}$, and express the equation of state in terms of these variables. Simultaneously plot the reduced equation of state, p vs v for three representative values of t ($t > 0$, $t = 0$ and $t > 0$).

Check out this page: `http://sgovindarajan.wikidot.com/notes:phase-transitions`