DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5460 Classical Field Theory Assignment 2 16.8.2010 (due: 23.8.2010)

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1. An arbitrary spatial rotation can be written as a rotation by an angle θ about an axis (specified by a unit vector $\hat{n}$). This is only true in three dimensions. Why?. Let us call the corresponding rotation matrix $R(\hat{n}, \theta)$. Obtain an explicit expression for this 3×3 matrix.

2. Let $x'^{\mu} = \Lambda^{\mu}{}_{\nu}\ x^{\nu} + a^{\mu}$ represent an arbitrary transformation under the Poincaré group. Here the matrix Λ parametrises Lorentz tranformations and a^{μ} parametrises translations in spacetime. Consider two successive such transformations with parameters (Λ_1, a_1) and (Λ_2, a_2) and obtain the composition rule. Verify that all the axioms of a group are satisfied.

3. An arbitrary Lorentz boost with rapidity, ϕ, along a direction $\hat{n}$ can be represented by a 4×4 matrix $B(\hat{n}, \phi)$. Obtain an explicit expression for $B(\hat{n}, \phi)$. In the limit of small rapidity, show that one recovers a Galilean boost along the direction $\hat{n}$.

4. A particle of mass m and charge q is at rest in a region is space with constant and uniform magnetic field $\mathbf{B} = B\ \hat{e}_z$. Work out the electromagnetic fields in an inertial frame moving with constant velocity $\mathbf{v} = v\ \hat{e}_x$ with respect to the particle. Show that the Lorentz force on the particle is zero in the moving frame.

5. Let Ω be the $2n \times 2n$ antisymmetric matrix, $\begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix}$, where I_n is the $n \times n$ identity matrix. A symplectic $2n \times 2n$ matrix S (with real entries) satisfies the symplectic condition
$$S \cdot \Omega \cdot S^T = \Omega\ .$$
Verify that the set of all such matrices form a group, called the symplectic group $Sp(2n, \mathbb{R})$. Writing $S = \exp(\lambda K)$, first obtain the condition(s) on the matrix K such that the symplectic condition holds to first order in λ. Next, writing $K = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ in terms of $n \times n$ matrices A, B, C and D, work out the condition(s) on these four matrices.

Recommended reading: V. Balakrishnan, *How is a vector rotated?*, Resonance, Vol. **4**, No. 10, pp. 61-68 (1999). Also available at the URL:
`http://www.physics.iitm.ac.in/%7Elabs/dynamical/pedagogy/`