

DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5460 Classical Field Theory Assignment 1 31.7.2012 (due: 7.8.2012)

1. A particle of mass m and charge q moves under the influence of an electromagnetic field. The electric and magnetic fields (not necessarily time-independent) can be written in terms of the scalar potential $\Phi(\mathbf{x})$ and vector potential $\mathbf{A}(\mathbf{x})$ as follows:

$$\mathbf{E} = -\nabla\Phi - \partial\mathbf{A}/\partial t, \quad \text{and} \quad \mathbf{B} = \nabla \times \mathbf{A} .$$

Show that the Euler-Lagrange equations for the Lagrangian given below reproduces the Lorentz force law:

$$L = \frac{1}{2}m \dot{\mathbf{x}}^2 + q \dot{\mathbf{x}} \cdot \mathbf{A}(\mathbf{x}) - q \Phi(\mathbf{x}) .$$

This is an example of a Lagrangian that is **not** of the form $T - V$ as it involves a velocity-dependent conservative force.

2. The Hamiltonian for a system with one degree of freedom has the form

$$H = \frac{p^2}{2a} - bqpe^{-\alpha t} + \frac{ba}{2}q^2e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{kq^2}{2} ,$$

where a , b , α , and k are constants.

- (a) Find a Lagrangian corresponding to this Hamiltonian.
 - (b) Find an equivalent Lagrangian that is not explicitly dependent on time.
 - (c) What is the Hamiltonian corresponding to the second Lagrangian, and what is its relationship between the two Hamiltonians?
3. The point of suspension of a simple pendulum of length l and mass m is constrained to move on a parabola $z = ax^2$ in the vertical $x - z$ plane. Derive a Hamiltonian governing the motion of the pendulum and its point of suspension. Obtain the Hamilton's equations of motion.

4. A free particle of unit mass moving in one dimension with coordinate q . We are given that the particle was $q = 1$ at $t = 0$ and at $q = 5$ at time $t = 1$. The action for this particle is

$$S[q(t)] = \frac{1}{2} \int_0^1 dt (\dot{q})^2 .$$

Consider the following four possible trajectories of the particles:

$$q_A(t) = 2 \sin(2\pi t) + 4t + 1 ,$$

$$q_B(t) = -3t^2 + 7t + 1 ,$$

$$q_C(t) = 4t + 1 ,$$

$$q_D(t) = -2 \cos(\pi t) + 3 .$$

Of the four paths, $q_C(t)$ is the solution to the equations of motion – this is usually called the *classical solution*. Compute the action for all four trajectories and verify that the minimum occurs for $q_C(t)$.

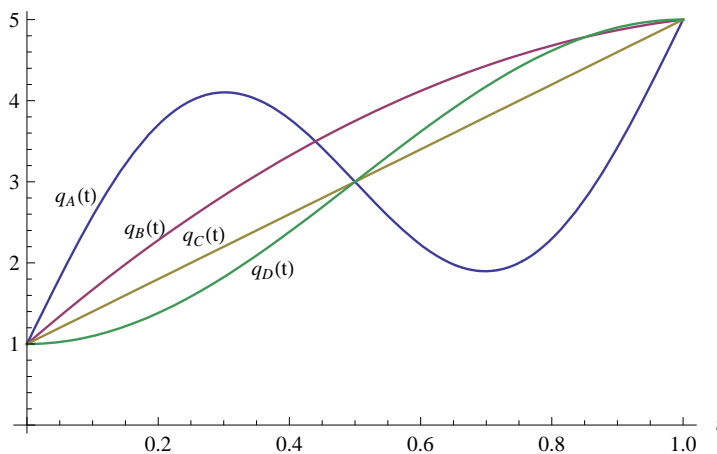


Figure 1: A plot of the four trajectories. Note that all paths begin at $q = 1$ and end at $q = 5$.

5. Consider the action for a particle of unit mass in a harmonic potential

$$S[q(t)] = \frac{1}{2} \int_0^1 dt \left[(\dot{q})^2 - \pi^2 (q - 3)^2 \right] .$$

Evaluate the action for the four trajectories that were considered in the previous problem. Show that the minimum value occurs for the solution $q_D(t)$ and verify that it is indeed the classical solution.