

**DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS**

PH5460 Classical Field Theory

Assignment 2

9.8.2012

Functional Calculus and Equations of Motion

1. Consider the following functionals of N scalar fields $\phi^i(x)$ (m and λ are constants)

$$T_0 = \int d^4x \sum_{a=1}^N \eta^{\mu\nu} (\partial_\mu \phi_a) (\partial_\nu \phi_a) \quad ,$$

$$U_0 = \int d^4x \left(m^2 \sum_{a=1}^N (\phi_a)^2 + \lambda \left[\sum_{a=1}^N (\phi_a)^2 \right]^2 \right) \quad ,$$

where $\mu = 0, 1, 2, 3$, $\eta^{\mu\nu} = \text{Diag}(1, -1, -1, -1)$ and $\partial_\mu = \partial/\partial x^\mu$. Evaluate the following:

$$(a) \frac{\delta \phi_a(x)}{\delta \phi_b(y)} \quad \text{and} \quad (b) \frac{\delta \partial_\mu \phi_a(x)}{\delta \phi_b(y)}$$

$$(c) \frac{\delta T_0}{\delta \phi_b(y)} \quad \text{and} \quad (d) \frac{\delta U_0}{\delta \phi_b(y)}$$

Using the result of the last two parts, obtain the equations of motion for the action: $S_0 = T_0 - U_0$.

2. Consider the following functionals of a vector field $A_\mu(x)$ (m is a constant of suitable dimensions)

$$T_1 = -\frac{1}{4} \int d^4x F_{\mu\nu} F_{\rho\sigma} \eta^{\mu\rho} \eta^{\nu\sigma} \quad ,$$

$$U_1 = m^2 \int d^4x A_\mu A_\nu \eta^{\mu\nu} + 4\pi \int d^4x J^\mu A_\mu \quad ,$$

where $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$ and J^μ is a four-current. Evaluate the following:

$$(a) \frac{\delta F_{\mu\nu}(x)}{\delta A_\rho(y)} \quad (b) \frac{\delta T_1}{\delta A_\rho(y)} \quad (c) \frac{\delta U_1}{\delta A_\rho(y)}$$

Using the result of the last two parts, obtain the equations of motion for the *Proca action*: $S_1 = T_1 - U_1$. Do you recognise the equations of motion in the $m = 0$ limit?