DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5460 Classical Field Theory   Assignment 6   3.9.2012 (due: 13.9.2012)

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1. Consider the following Lagrangian density in 1+1 dimensions

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{2}(\phi^2 - a^2)^2 \quad .$$

   (a) Obtain the Euler-Lagrange equation corresponding to the Lagrangian density. Verify that the kink soliton given by

$$\phi(x) = a\tanh\mu x \quad ,$$

   is a solution of this equation for a particular value of μ.

   (b) Obtain the stress tensor corresponding to the above Lagrangian density.

   (c) Plot the variation of the Hamiltonian density in space for the kink soliton. In the same plot, plot $\phi(x)$ for the kink soliton.

   (d) For the kink soliton, calculate $E = \int dx\ \mathcal{H}$ and $P = \int dx\ T^{01}$.

   (e) By boosting the solution corresponding to the kink soliton, we obtain a new solution of the Euler-Lagrange equation. Verify that this is true. For this solution, now calculate E and P. Calculate $(E^2 - P^2)$ and compare with the unboosted solution.

2. Consider N real scalar fields ϕ_i $(i = 1, \ldots, N)$ whose Lagrangian density is given by

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{N}(\partial_\mu\phi_i\partial^\mu\phi_i) - V(\rho) \quad ,$$

   where $\rho \equiv \sum_i \phi_i^2$. Show that the action is invariant under infinitesimal global $SO(N)$ transformations:

$$\delta\phi_i = \sum_j \theta_{ij}\phi_j \quad ,$$

   where $\theta_{ij} = -\theta_{ji}$ are real numbers. Obtain the Noether currents associated with this symmetry and verify that they are indeed conserved.