Department of Physics
Indian Institute of Technology Madras
PH5480 Quantum Field Theory Problem Set 2 13.2.2013

An arbitrary complex solution of the massive Klein-Gordon equation (with mass m), $\psi(x)$ can be expanded as follows

$$\psi(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} \left(b(\mathbf{k})\ e^{-ik\cdot x} + c^\dagger(\mathbf{k})\ e^{ik\cdot x} \right)$$

with $k = (\omega_k, \mathbf{k})$ and $\omega_k = \sqrt{m^2 + \mathbf{k}\cdot\mathbf{k}}$. In the quantum theory, assume the following commutation relations

$$[b(\mathbf{k}), b^\dagger(\mathbf{k}')] = \delta^3(\mathbf{k}-\mathbf{k}') \quad \text{and} \quad [c(\mathbf{k}), c^\dagger(\mathbf{k}')] = \delta^3(\mathbf{k}-\mathbf{k}')\ ,$$

with other commutators vanishing.

1. Show that the ETCR between ψ and its conjugate momentum (as well as similar relations for ψ^* and its conjugate momentum) hold. Also, verify that the $[\psi, \psi]$ and other such ETCR's vanish.

2. Obtain expressions for the Hamiltonian for this system in terms of b, $b^\dagger$ and so on as was done for a real scalar field.

3. Finally, evaluate the commutation relations

$$[\psi(x), \psi(y)] \quad , \quad [\psi(x), \psi^*(y)] \quad , \quad [\psi^*(x), \psi^*(y)]\ ,$$

   and express your answers in terms of known Green functions.

4. Also, compute

$$\langle 0|T(\psi(x)\psi(y))|0\rangle \quad , \quad \langle 0|T(\psi(x)\psi^*(y))|0\rangle \quad , \quad \langle 0|T(\psi^*(x)\psi^*(y))|0\rangle\ .$$