**DEPARTMENT OF PHYSICS**
**INDIAN INSTITUTE OF TECHNOLOGY, MADRAS**

PH5460 Classical Field Theory    Assignment 2    8.8.2014 (due: 14.8.2014)

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*Functional Calculus and Equations of Motion*

1. Consider the following functionals of $N$ scalar fields $\phi^i(x)$ ($m$ and $\lambda$ are constants)

$$\begin{aligned} T_0 &= \int d^4x \sum_{a=1}^{N} \eta^{\mu\nu}(\partial_\mu \phi_a)(\partial_\nu \phi_a) \quad , \\ U_0 &= \int d^4x \left( m^2 \sum_{a=1}^{N} (\phi_a)^2 + \lambda \left[ \sum_{a=1}^{N} (\phi_a)^2 \right]^2 \right) \quad , \end{aligned}$$

where $\mu = 0, 1, 2, 3$, $\eta^{\mu\nu} = \text{Diag}(1, -1, -1, -1)$ and $\partial_\mu = \partial / \partial x^\mu$. Evaluate the following:

$$(a) \quad \frac{\delta \phi_a(x)}{\delta \phi_b(y)} \quad \text{and} \quad (b) \quad \frac{\delta \partial_\mu \phi_a(x)}{\delta \phi_b(y)}$$
$$(c) \quad \frac{\delta T_0}{\delta \phi_b(y)} \quad \text{and} \quad (d) \quad \frac{\delta U_0}{\delta \phi_b(y)}$$

Using the result of the last two parts, obtain the equations of motion for the action: $S_0 = T_0 - U_0$. Obtain the energy density for this Lagrangian.

2. Consider the following functionals of a vector field $A_\mu(x)$ ($m$ is a constant of suitable dimensions)

$$\begin{aligned} T_1 &= -\frac{1}{4} \int d^4x \ F_{\mu\nu} F_{\rho\sigma} \eta^{\mu\rho} \eta^{\nu\sigma} \quad , \\ U_1 &= m^2 \int d^4x \ A_\mu A_\nu \eta^{\mu\nu} + 4\pi \int d^4x \ J^\mu A_\mu \quad , \end{aligned}$$

where $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$ and $J^\mu$ is a four-current. Evaluate the following:

$$(a) \quad \frac{\delta F_{\mu\nu}(x)}{\delta A_\rho(y)} \qquad (b) \quad \frac{\delta T_1}{\delta A_\rho(y)} \qquad (c) \quad \frac{\delta U_1}{\delta A_\rho(y)}$$

Using the result of the last two parts, obtain the equations of motion for the *Proca action*: $S_1 = T_1 - U_1$. Do you recognise the equations of motion in the $m = 0$ limit? Obtain the energy density for this Lagrangian.