

DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5460 Classical Field Theory Assignment 6 10.9.2014 (due: 17.9.2014)

Consider the following Lagrangian density in 1+1 dimensions

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{2} (\phi^2 - a^2)^2 \quad .$$

1. Obtain the Euler-Lagrange equation corresponding to the Lagrangian density. Verify that the kink soliton given by

$$\phi(x) = a \tanh \mu x \quad ,$$

is a solution of this equation for a particular value of μ .

2. Obtain the stress tensor corresponding to the above Lagrangian density.
3. Plot the variation of the Hamiltonian density in space for the kink soliton. In the same plot, plot $\phi(x)$ for the kink soliton.
4. For the kink soliton, calculate $E = \int dx \mathcal{H}$ and $P = \int dx T^{01}$.
5. By boosting the solution corresponding to the kink soliton, we obtain a new solution of the Euler-Lagrange equation. Verify that this is true. For this solution, now calculate E and P . Calculate $(E^2 - P^2)$ and compare with the result that you obtained for the unboosted solution.