**DEPARTMENT OF PHYSICS**
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**PH5211 High Energy Physics** **Problem Set 1** **13.8.2014**

This problem is mostly about the kinematics of energy conservation in physical processes. We shall begin with the decay of a particle of mass $M$ and three-momentum $\mathbf{P}$ into $n$ decay products with masses $m_i$, and three-momenta $\mathbf{p}_i$ with $i = 1, 2, \ldots, n$. In special relativity, the conservation of four-momentum is given by

$$P^\mu = \sum_{i=1}^{n} p_i^\mu \quad , \quad \mu = 0, 1, 2, 3 \ ,$$

where $P^\mu = (E/c, \mathbf{P})$ is the four-momentum of the initial particle and $p_i^\mu = (E_i/c, \mathbf{p}_i)$ represents the four-momentum of the $i$-th decay product.

1. Express $E$ in terms of $\mathbf{P}$ and the mass $M$. Show that the integration measure $\frac{d^3P}{(2\pi)^3 2E}$ is Lorentz invariant.
2. **Two-body decays:** For two decay products, in the rest frame of the particle of mass $M$, show that
$$\frac{E_1}{c^2} = \frac{M^2 - m_2^2 + m_1^2}{2M} \quad ,$$
$$\frac{|\mathbf{p}_1|}{c} = \frac{|\mathbf{p}_2|}{c} = \frac{\left[(M^2 - (m_1 + m_2)^2)(M^2 - (m_1 - m_2)^2)\right]^{1/2}}{2M} \ .$$
3. **Three-body decays:** In the rest frame of the particle of mass $M$:
   (a) Show that the conservation of three-momentum implies that the three decay products **must** lie in a plane.
   (b) Define the Lorentz scalars, $m_{ij}^2 := (p_i + p_j)^2/c^2$. Show that $m_{ij}^2 > (m_i + m_j)^2$. *Hint:* Work in the rest frame of one of the particles, say the $i$-th one amd compute $(p_i + p_j)^2$.
   (c) Show that $m_{12}^2 = M^2 + m_3^2 - (2ME_3)/c^2$ and hence $m_{12}^2 \leq (M - m_3)^2$.
   (d) Also show that $m_{12}^2 + m_{23}^2 + m_{13}^2 = M^2 + m_1^2 + m_2^2 + m_3^2$.
4. The beta decay of the neutron. Here $M = m_n = 939.56533(4)$ MeV/$c^2$, $m_1 = m_e = 0.510998902(21)$ MeV/$c^2$, $m_2 = m_p = 938.27200(4)$ MeV/$c^2$ and $m_3 = m_{\nu_e} \leq 3eV$. Let us assume that neutron is at rest.
   (a) Show that the maximum electron energy is
$$E_e^{\max} = \frac{m_n c^2}{2}\left(1 + \frac{m_e^2}{m_n^2} - \frac{(m_p + m_{\nu_e})^2}{m_n^2}\right) = E_{e,0}^{\max} - m_{\nu_e}\frac{m_p}{m_n} + O(m_{\nu_e}^2) \ .$$
   where $E_{e,0}^{\max}$ is the maximum electron energy if the neutrino mass were zero. Show that the maximum kinetic energy $T_e := E_{e,0}^{\max} - m_e c^2 = 0.7815799$ MeV. Is such an electron relativistic?
   (b) Show that the maximum proton energy is
$$E_p^{\max} = \frac{m_n c^2}{2}\left(1 + \frac{m_p^2}{m_n^2} - \frac{(m_e + m_{\nu_e})^2}{m_n^2}\right) = E_{e,0}^{\max} - m_{\nu_e}\frac{m_e}{m_n} + O(m_{\nu_e}^2) \ .$$
   where $E_{p,0}^{\max}$ is the maximum electron energy if the neutrino mass were zero. Show that $T_p := E_{p,0}^{\max} - m_p c^2 = 0.00075118918$ MeV. Is such a proton relativistic?