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PH1020 Physics II **Problem Set 2** **15.1.2014**

The Electric field and its flux

1. Consider a spherical surface of radius R centred at the origin. Since points on the surface are at fixed radius, $\delta r = 0$ for any two points on the sphere. The area of an infinitesimal spherical rectangle with corners (θ, ϑ) and $(\theta + \delta\theta, \varphi + \delta\varphi)$ is
$$h_\theta h_\varphi \delta\theta\delta\varphi = R^2 \sin\theta\delta\theta\delta\varphi := R^2\delta\Omega\ .$$
One says that the spherical rectangle *subtends* a **solid angle** $\delta\Omega$ at the point O. More generally, one considers the projection of any object onto the surface of a unit sphere about the point of interest. The area of the projection is the solid angle of the object. The SI unit of solid angle is **steradians** and the total solid angle of a sphere about any point in its interior is 4π. Solid angles and more generally, spherical trigonometry play an important role in observational astronomy.

   (a) Consider a point charge, q at a point P.. Show that the electrical flux through a region, D, on the surface of a sphere centred at P is $q/(4\pi\epsilon_0)$ times the solid angle subtended by D at P. Now consider a cube centred at P. What is the electrical flux through a face of the cube? Does it depend on the size of the cube? Does it depend on the orientation of the cube?

   (b) Two point charges q and $-q$ are separated by a distance $2a$. Find the flux of the electric field through a disc, D with boundary a circle C of radius R, placed at the mid-point between the charges, with its plane perpendicular to the line joining them.

2. (a) A sphere of radius R centred at the origin carries a *surface* charge density $\sigma(\mathbf{r}) = (\mathbf{K} \cdot \mathbf{r})$, where $\mathbf{K}$ is a given constant vector of appropriate dimensions. Find the electric field at the centre of the sphere.

   (b) Repeat the calculation for the case in which the sphere has a *volume* charge density $\rho(\mathbf{r}) = (\mathbf{K} \cdot \mathbf{r})$ $(0 \leq |\mathbf{r}| \leq R)$, rather than a surface charge density. (Again, $\mathbf{K}$ has the appropriate physical dimensions.)

3. Using Gauss' law in the integral form,
$$\oint_S \mathbf{E} \cdot \hat{n} dS = \frac{Q_{\text{enclosed}}}{\epsilon_0}\ ,$$
where $\hat{n}$ is the outward normal to the Gaussian surface S, obtain the electric field $\mathbf{E}$ due to the following volume charge distributions:

(a)

$$\rho(\varrho, \varphi, z) = \begin{cases} \beta\varrho/a & 0 < \varrho \leq a \\ 0 & a < \varrho < \infty \ . \end{cases}$$

Here (ϱ, φ, z) denote cylindrical polar coordinates. Using cylindrical symmetry one can show that $\mathbf{E}(\mathbf{r})$ must necessarily be of the form $E(\varrho)\hat{e}_\varrho$. So the problem reduces to a choice of Gaussian surface, S.

(b)

$$\rho(r, \theta, \varphi) = \begin{cases} \beta[1 - (r^2/a^2)] & 0 < r \leq a \\ 0 & a < r < \infty \ . \end{cases}$$

Here (r, θ, φ) denote spherical polar coordinates. Using spherical symmetry one can show that $\mathbf{E}(\mathbf{r})$ must necessarily be of the form $E(r)\hat{e}_r$.

Note: β is a constant of appropriate dimensions in each case.

4. A ring of radius R and uniform charge density λ is located in the yz-plane, with its centre at the origin O. A particle of mass m and charge $-Q$ is constrained to move along the x-axis. Show that the time period of oscillation of the particle, when it is displaced slightly from O along the x-axis and released, is

$$T = 2\pi\sqrt{\frac{2\epsilon_0 m R^2}{\lambda Q}} \quad .$$

5. Two infinite lines of charge, each of uniform charge density λ, are located along the x and y axes respectively. Consider a cubical Gaussian surface with edge length L, centred at the origin of the coordinates O with its faces perpendicular to the coordinate axes. Find the electric flux through *each* of the six faces of the cube.