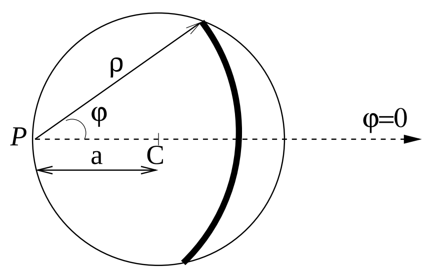
DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH1020 Physics II **Problem Set 3** **22.1.2014**

Potential Problems

1. A circular sheet of radius a has uniform surface charge density σ. Calculate the potential at a point P on the circumference of the sheet, and also at C, the centre of the circle.
   Hint: Taking P as the origin and PC as the line corresponding to $\varphi = 0$, the equation of the circle in polar coordinates is $\varrho = 2a\cos\varphi$, where $-\pi/2 \leq \varphi \leq \pi/2$. Take elementary strips of area $2\varrho\varphi d\varrho$ as shown in the figure in order to evaluate the potential at P. Choose the limits of integration carefully.



2. Evaluate the electrostatic energy W of a charge distribution in the form of a uniform charge density within a sphere of radius a and total charge Q. (Your answer must be in terms of ϵ_0, a and Q.)

3. The computation of the electrostatic potential $\phi(\mathbf{r})$ reduces to solving Laplace's equation, $\nabla^2\phi = 0$ subject to boundary conditions. Consider the following situations:

   (a) Consider a situation with cylindrical symmetry as well as translational symmetry along the z-direction. In such cases, the potential is independent of the coordinates φ and z i.e., $\phi = \phi(\varrho)$.. Show that the most general solution to Laplace's equation in this case takes the form $\phi = A\log\varrho + B$ for suitable constants A and B. Determine A and B using the following boundary conditions:

   $$\phi(\varrho)\big|_{\varrho=a} = V_0 \quad \text{and} \quad \phi(\varrho)\big|_{\varrho=b} = 0 \quad ,$$

   for $a > b > 0$. Take the region of interest to be $b \leq \varrho \leq a$. Compute the electric field for this electrostatic potential. Plot lines of equipotential as well as electrical lines of force.

   (b) Consider the situation where two (infinite) metal plates are parallel to each other and separated by a distance d. One plate is grounded and the other is kept at V_0. Choose the direction of the separation to be the y-axis. Solve Laplace's equation in the region between the two plates subject to the given boundary conditions. Compute the electric field for this electrostatic potential. Plot lines of equipotential as well as electrical lines of force.

(c) Consider a grounded (i.e., at zero potential) infinitely long metal sheet (equipotential surface) lying in the $z = 0$ plane. A point charge $+Q$ is located at $z = a > 0$. Verify that

$$\phi(x, y, z) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{\sqrt{x^2 + y^2 + (z-a)^2}} - \frac{Q}{\sqrt{x^2 + y^2 + (z+a)^2}}\right) \quad ,$$

solves Laplace's equation at all points in the region $z \geq 0$ except for the location of the charge as well as satisfying the boundary conditions. If possible, interpret the second term in the above expression in terms of an image charge located somewhere.

4. The electric field in a certain region in space is given by

$$\mathbf{E}(x, y, z) = \frac{k}{\epsilon_0 a^2}\left(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z\right) \quad ,$$

where k and a are constants with appropriate physical dimensions. Is the field conservative? If so, calculate the electrostatic potential as well as the charge density at an arbitrary point $P(x, y, z)$.

5. Prove the **Mean-Value Theorem:** If S_R is the surface of a (mathematical) sphere of radius R whose interior contains no charge, then the potential at the center is equal to the average potential over the surface S_R. Show this by establishing that the average is independent of the radius of the sphere.

6. A spherically symmetric charge distribution (density depends only on the radial coordinate) is placed in a given electric field produced by charges external to the spherical distribution. Prove that the net force experienced by the spherical distribution is as though all of the charge were concentrated at the center of the distribution. (*Hint:* Use the mean-value theorem.)