DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH1020 Physics II **Problem Set 5** **5.2.2014**

Mostly Conductors

1. Consider a spherical conducting shell of volume V. Examine whether the following statements are **true** or **false**, and justify your answer:
   (a) "The region inside the shell is totally shielded from the influence of charges placed outside the shell."
   (b) "The region outside the shell experiences an electric field when a charge is placed inside the shell, and this field depends on the actual location of the charge inside the shell."
   (c) "The shell should be earthed if the region outside it is to be shielded from charges placed within it."

2. Two parallel infinite conducting plates lying at $x = 0$ and $x = L$ have potentials ϕ_0 and 0 respectively. Using Poisson's equation with the appropriate boundary conditions, find (a) the electric field between the plates and (b) the surface charge densities on the plates, when the free volume charge density between the plates is equal to a nonzero constant k.

3. Two charges $+q$ and $-2q$ are at $(0, 0, 3d)$ and $(0, 0, d)$ respectively. If the $x - y$ plane is a grounded conductor, find the force on the charge $+q$.

4. A point charge q is at the centre of an uncharged conducting spherical shell of finite thickness, with inner and outer radii a and b respectively. Find the work done on the system when q is removed from its original position to a very large distance from the conducting shell, through a small hole in it.

5. An isolated infinite conducting plane has a constant surface charge density σ. The normal to the plane is along the z axis. A hemispherical Gaussian surface of radius R whose centre is on the z axis, at a distance $2R$ from the plane, has its symmetry axis at an angle α to the z axis. Find the electric flux through the curved surface of the hemisphere.

6. Prove the following identity for any vector field $\mathbf{P}(\mathbf{r})$:
$$\int_V dV\, \mathbf{P}(\mathbf{r}) = \int_S \mathbf{r}\, [\mathbf{P}(\mathbf{r}) \cdot \mathbf{dS}] - \int_V dV\ \mathbf{r}\ [\boldsymbol{\nabla} \cdot \mathbf{P}(\mathbf{r})]\ ,$$
   where V is the volume enclosed by the surface S.
   Hint: Take the dot product of the above identity with a constant vector $\mathbf{a}$ to obtain a new identity. Prove this new identity and then argue that it implies the required identity.

7. A point dipole of moment $\mathbf{p} = p_0(\hat{e}_x + 2\hat{e}_y + 3\hat{e}_z)$ is placed in an electrostatic potential ϕ given by
$$\phi(x, y, z) = \phi_0 \left[1 + \frac{(x^2 + y^2 + z^2)}{a^2} + \frac{(x^4 + y^4 + z^4)}{a^4} \right]$$
   where ϕ_0 and a are appropriate constants. Find the force and the couple acting on the dipole when it is located at the point (a, a, a). Find also the torque of the force about the origin.