

DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH1020 Physics II

Problem Set 6

10.2.2014

Simply Dielectrics

1. A dielectric medium carries a *uniform* polarization \mathbf{P}_0 . A spherical cavity is scooped out inside the medium. Find (i) the bound surface charge density on the surface of the cavity, and (ii) the electric field at the centre of the cavity due to this surface charge.
2. A cylinder of length $2L$ and radius a is centred at the origin, with the z -axis as its symmetry axis. The cylinder is uniformly polarized with polarization $\mathbf{P} = P_0 \hat{e}_z$, where P_0 is a constant.
 - (a) Find the bound charge densities ρ_b and σ_b .
 - (b) Find the electric field at all points on the positive z -axis, and verify that it satisfies the appropriate boundary condition at $z = L$.
 - (c) Find the electric field at the origin, and sketch its magnitude as a function of the ratio a/L .
3. Consider a uniform spherical free charge distribution of radius a and charge density ρ_0 . This region is filled with a medium of dielectric constant κ_1 , and surrounded by a medium of dielectric constant κ_2 . Find (i) the bound volume charge density everywhere in space, and (ii) the bound surface charge density on the surface of the sphere of radius a .
4. A sphere of radius R and dielectric constant κ , centred at the origin of coordinates, is placed in a constant electric field \mathbf{E}_0 directed along the z axis. The corresponding electrostatic potential is given by $\phi(r, \theta, \varphi) = (-E_0 r + b_1 r^{-2}) \cos \theta$ outside the sphere, and $\phi(r, \theta, \varphi) = b_2 r \cos \theta$ inside the sphere. Find
 - (a) the constants b_1 and b_2 , in terms of κ , E_0 and R ;
 - (b) the electric field at all points in space;
 - (c) the polarization \mathbf{P} of the sphere, and the dipole moment of the sphere about the origin;
 - (d) the volume and surface densities of the bound charge in the sphere.
5. Three identical metallic plates (of non-zero thickness) are kept vertical and parallel to each other. The two outer plates are separated from the central plate by distances a and b , respectively. The outer plates are connected by a thin conducting wire and a charge $+Q$ is placed on the central plate. Find the charge on all the six vertical surfaces, neglecting edge effects.
6. A point charge q is on the plane separating vacuum from an infinite homogeneous dielectric with dielectric constant κ . Find the magnitudes of the displacement vector and the electric field in the entire space.