DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH1020 Physics II **Problem Set 8** **10.3.2014**

1. **Force between two magnetic dipoles**

   (a) The vector potential due to a point magnetic dipole $\mathbf{m}$ located at the origin has been shown to be $\mathbf{A}_{\rm dipole}(\mathbf{r}) = (\mu_0/4\pi)(\mathbf{m} \times \hat{e}_r)/r^2$. Find the magnetic field $\mathbf{B}(\mathbf{r})$ due to a point magnetic dipole located at the origin.

   (b) The force on a magnetic (point) dipole $\mathbf{m}$ placed in a magnetic field $\mathbf{B}(\mathbf{r})$ is given, in analogy with the expression already known to us from electrostatics, by $\mathbf{F}(\mathbf{r}) = (\mathbf{m} \cdot \nabla)\mathbf{B}(\mathbf{r})$. (This expression is valid if there is no external current at the point $\mathbf{r}$.) Use this to find the force exerted on each other by two magnetic point dipoles $\mathbf{m}_1$ and $\mathbf{m}_2$ that are located, respectively, at the origin and at $(0, 0, d)$ where $d > 0$, with their directions along the positive z-axis.

2. A spherical shell of radius R carrying uniform surface charge density σ rotates with constant angular speed ω about a diameter.

   (a) Show that the magnetic field inside the shell is uniform.

   (b) Using dimensional analysis, obtain an expression for the magnitude of the magnetic force of attraction between the northern and southern hemispheres, up to a numerical factor.

3. **The Hall Effect**: The Hall effect is an important experimental method for finding both the **sign** and the magnitude of the charge on the particles carrying electric current in a medium.
   Consider a conducting material with rectangular cross-section PQRS, as shown in the figure below. A potential difference is applied between the top and bottom faces of the slab, so as to produce a uniform current flow $-I\,\hat{e}_z$ across the cross-section. Conduction electrons (current carriers) therefore move with a drift velocity $\mathbf{v} = +v_0\,\hat{e}_z$. The conductor is placed in a uniform transverse magnetic field $\mathbf{B} = B_0\,\hat{e}_y$.

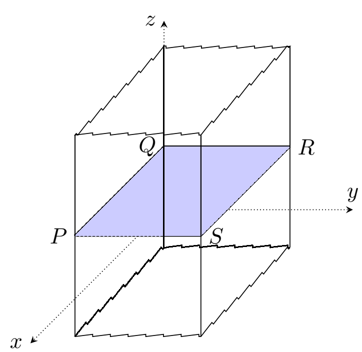

Figure 1: The configuration for the Hall effect

   (a) In which direction do the electrons get deflected?
   (This deflection results in an accumulation of unlike charges on the opposite faces of the slab containing PS and QR respectively, which in turn produces an electrical force to counteract the magnetic force. This accumulation of charges continues till the two forces exactly cancel.)

   (b) Find the resulting potential difference (the Hall voltage) between the side faces of the slab normal to the y-direction, in terms of the electronic charge e, B_0 and the relevant dimensions of the slab.

   (c) How would your analysis change if the moving carriers were positively charged?

4. This problem is related to Thomson's classic method for measuring the e/m ratio of a charged particle.

   A particle of charge $+q$ and mass m is located at the origin O at time $t = 0$ (see figure below). A constant electric field $\mathbf{E} = E_0\,\hat{e}_y$ is present in the region between $x = 0$ to $x = L$, between the plates of a parallel plate capacitor. The initial velocity of the particle is $v_0\,\hat{e}_x$ with $v_0 > 0$. The particle leaves the region between the capacitor plates and strikes a screen (located at $x = D + (L/2)$) at the point R$'$. Neglect the end effects of the capacitor.

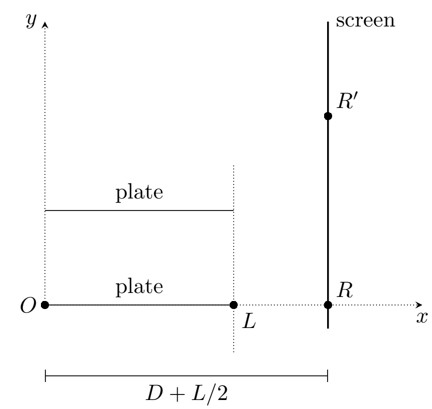

Figure 2: The Thomson experiment

   (a) Find the distance RR$'$, i.e., the vertical deflection suffered by the particle.

   (b) Sketch the trajectory of the particle in the xy-plane.

5. A point magnetic dipole is placed at the origin of coordinates O, with its dipole moment $\mathbf{m}$ along the positive z-axis. Imagine a surface in the form of a right circular cylinder of height $2L$ and radius L, with its plane faces perpendicular to the z-axis and centred at O. Find the flux of the magnetic field $\mathbf{B}$ through (i) the plane top surface of the cylinder, and (ii) the curved surface of the cylinder.