DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5020 Electromagnetic Theory — Problem Set 2 — 26 Jan. 2016
(Will be discussed on **January 29, 2016**)

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1. The time-averaged potential of a neutral hydrogen atom is given by

$$\Phi = \frac{q}{4\pi\epsilon_0} \frac{e^{-\alpha r}}{r} \left(1 + \frac{\alpha r}{2}\right)$$

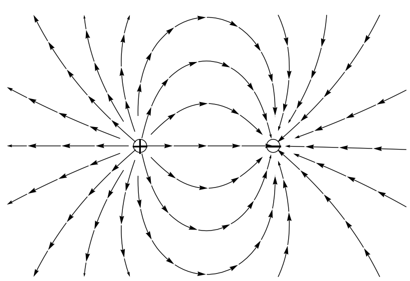
where q is the magnitude of the electronic charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

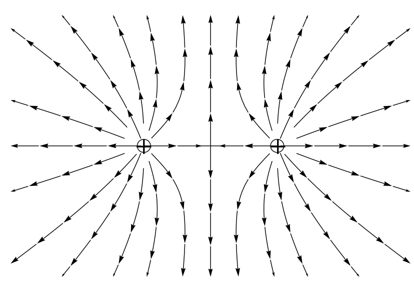
2. A spherically symmetric charge distribution of radius R has a charge density given by

$$\rho(r) = \begin{cases} \rho_0 \left(1 - \frac{r}{R}\right) & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

Find the electric field as a function of r.

3. Graphically, using your favourite program/plotting routine, draw representative electric fields lines for (i) a pair of equal and opposite charges and (ii) for a pair of identical charges as shown below.





Hint: Let $\psi(\mathbf{x})$ = constant represent the curve for a line of force. then $\mathbf{E} \cdot \nabla\psi = \nabla\Phi \cdot \nabla\psi = 0$ along the curve. Find such a $\psi(\mathbf{x})$ and plot it to get the lines of force.

4. Derive the two dimensional form of Green's boundary value theorem: if $\phi(x, y)$ is the two dimensional potential, show that

$$\Phi(\mathbf{r}) = \frac{1}{2\pi\epsilon_0} \int_S G(\mathbf{r}, \mathbf{r}')\rho(\mathbf{r}')d^2r' + \frac{1}{2\pi} \int_C \Big(G(\mathbf{r}, \mathbf{r}')\nabla'\phi - \phi\nabla' G(\mathbf{r}, \mathbf{r}')\Big) \cdot \hat{n}\, dl\,,$$

where S is the area bounded by the contour C and $\hat{n}$ is the outward normal unit vector.

5. Consider a potential problem in the half-space defined by $z \geq 0$, with Dirichlet boundary conditions on the plane $z = 0$ and at infinity.

(a) Write the Green function $G(\mathbf{x}, \mathbf{x}')$ suitable for this problem.

(b) If the potential on the $z = 0$ plane is given to be $\Phi = V$ inside a circle of radius a centred at the oriigin, and $\Phi = 0$ outside the circle, find an integral expression for the potential at a point P (located in the region $z > -0$) with coordinates (ϱ, φ, z) in cylindrical polar coordinates.

(c) Show that, along the axis of the circle ($\varrho = 0$), the potential is given by

$$\Phi = V \left(1 - \frac{z}{\sqrt{a^2 + z^2}} \right) .$$

(d) Show that at large distances such that $\varrho^2 + z^2 \gg a^2$, the potential can be expanded as a power series in $(\varrho^2 + z^2)^{-1}$, and the leading terms are

$$\Phi = \frac{Va^2}{2} \frac{z}{(\varrho^2 + z^2)^{3/2}} \left[1 - \frac{3a^2}{4(\varrho^2 + z^2)} + \frac{5(3\varrho^2 a^2 + a^4)}{8(\varrho^2 + z^2)^3} + \cdots \right]$$

(e) Verify that the results of parts (c) and (d) are compatible in the common region of vailidity.