

DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5020 Electromagnetic Theory

Problem Set 7

Mar 11, 2016

(Will be discussed on **March 14, 2016**)

1. (a) The electrostatic potential due to a point electric dipole with moment \mathbf{p} (located at the origin) is given by

$$\Phi_{\text{dipole}} = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{e}}_r}{r^2},$$

Find the electric field, \mathbf{E} , due to the dipole for an arbitrary orientation of the dipole moment \mathbf{p} .

Hint: Let the spherical polar coordinates of the vectors \mathbf{r} and \mathbf{p} be (r, θ, φ) and (p, θ_p, φ_p) respectively. Then the cosine of the angle between these two vectors is given by

$$\cos \alpha = \cos \theta \cos \theta_p + \sin \theta \sin \theta_p \cos (\varphi - \varphi_p).$$

- (b) The vector potential due to magnetic dipole with moment, \mathbf{m} , (located at the origin) is given by

$$\mathbf{A}_{\text{dipole}} = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{e}}_r}{r^2}.$$

Find the magnetic field, \mathbf{B} , due to the magnetic dipole.

- (c) Compare and comment on the two answers.
2. Find the exact magnetic field a distance z about the centre of a square loop of side s carrying current I . Verify that it reduces to the field of a magnetic dipole, with the appropriate magnetic dipole moment, when $z \gg s$.
3. A flat circular disc of radius R lies in the xy -plane with its centre at the origin. The disc has a uniform surface charge density $-\sigma$ from its centre up to a radius $R/\sqrt{2}$, and a uniform surface charge density $+\sigma$ for radii from $R/\sqrt{2}$ up to R . The disc rotates with a constant angular velocity ω about the z -axis. Show that the disc (whose total charge is zero) has a magnetic dipole moment directed along the z -axis, with magnitude equal to $\pi\sigma\omega R^4/8$.