National Programme on Technology Enhanced Learning*

http://nptel.iitm.ac.in/courses/115106058/

Physics – Classical Field Theory **Problem Set 4**

http://sgovindarajan.wikidot.com/cftontheweb

(To be solved while/after viewing lectures 6 and 7)

1. Explicitly write out the multiplication tables (usually called the *Cayley Table*) for the groups, $\mathbb{Z}_2 \times \mathbb{Z}_2$ and $\mathbb{Z}_4$, both of order 4. Hence, show that they are **not** isomorphic to each other.

2. The dihedral group, D_{2n}, is generated by two elements x and y satisfying the relations

$$x^n = e, \quad y^2 = e, \quad \text{and} \quad yx = x^{n-1}y \; . \qquad (*)$$

   (a) By working out the multiplication table for D_3, prove the isomorphism, $D_6 \sim S_3$.

   (b) Obtain subgroups, with order 2 and 3, of S_3 and check to see if they are *normal* subgroups. If they are normal, obtain the group multiplication law for the corresponding coset. Is S_3 a simple group?

3. Consider the cyclic group $\mathbb{Z}_3 = (e, g, g^2)$ with $g^3 = e$. The regular representation is a map from the group to 3×3 matrices with g being represented by the permutation matrix

$$g \longrightarrow M(g) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \; ,$$

   with the property that $M(g_1)M(g_2) = M(g_1 \cdot g_2)$ for any $g_1, g_2 \in \mathbb{Z}_3$.

   (a) What does the identity $g^3 = e$ become in this representation? Verify that it holds.

   (b) Obtain the similarity transformation that diagonalises $M(g)$.

4. Work out all finite groups with order ≤ 10.
   Further reading: Chap. 1 of J. S. Milne, *Group Theory*, available at http://www.jmilne.org/math/CourseNotes/gt.html.
   Milne lists all finite groups with order ≤ 24. Why does his list not contain groups of prime order?

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