DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5030 Classical Mechanics **Problem Set 10** **23.10.2021**

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1. Consider the following Lagrangian (of a double pendulum) with generalized coordinates (θ_1, θ_2).

$$L = \frac{1}{6} m L^2 \left(4\dot{\theta}_1^{\,2} + \dot{\theta}_2^{\,2} + 3\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2)\right) + \frac{1}{2} m g L \left(3\cos\theta_1 + \cos\theta_2\right)$$

   (a) Determine the matrices $\mathbb{T}$ and $\mathbb{V}$ that appear on expanding the above Lagrangian about the fixed point at $(\theta_1, \theta_2) = (0, 0)$.

   (b) Solve for the eigenvalues, $(\omega_1, \omega_2, \omega_3)$ of the equation

$$\det(-\omega^2 \mathbb{T} + \mathbb{V}) = 0 \ .$$

   (c) Obtain the normal mode for each eigenvalue i.e., the solution to the equation

$$\left(-\omega_i^2 \mathbb{T} + \mathbb{V}\right) \cdot \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} = 0 \ .$$

2. Let $\mathbb{T}$ and $\mathbb{V}$ denote the following two matrices.

$$\mathbb{T} = \begin{pmatrix} m & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & m \end{pmatrix} \quad , \quad \mathbb{V} = \begin{pmatrix} K & -K & 0 \\ -K & 2K & -K \\ 0 & -K & K \end{pmatrix} \ .$$

   (a) Solve for the eigenvalues, $(\omega_1, \omega_2, \omega_3)$ of the equation

$$\det(-\omega^2 \mathbb{T} + \mathbb{V}) = 0 \ .$$

   (b) Obtain the normal mode for each eigenvalue i.e., the solution to the equation

$$\left(-\omega_i^2 \mathbb{T} + \mathbb{V}\right) \cdot \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} = 0 \ .$$

3. Let $\mathbb{T}$ and $\mathbb{V}$ denote the following two matrices.

$$\mathbb{T} = \begin{pmatrix} (m_1 + m_2)\ell_1^2 & m_2 \ell_1 \ell_2 \\ m_2 \ell_1 \ell_2 & m_2 \ell_2^2 \end{pmatrix} \quad , \quad \mathbb{V} = \begin{pmatrix} (m_1 + m_2) g \ell_1 & 0 \\ 0 & m_2 g \ell_2 \end{pmatrix} \ .$$

   Obtain the eigenvalues for the equation $\det(-\omega^2 \mathbb{T} + \mathbb{V}) = 0$ and determine the corresponding normal modes.