DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5030 Classical Mechanics **Problem Set 2** **12.8.2021**

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1. Analyse the fixed points and their stability for the one-dimensional autonolous dynamical systems, $dx/dt = f(x)$:
(i) $f(x) = x(x^2 - 1)$, (ii) $f(x) = x(x^2 + 1)$ and (iii) $f(x) = (x^2 - 1)^2$.

2. Draw phase portraits for a particle moving in one-dimensional under a conservative force arising from the potentials.
(i) $V(x) = x(x^2 - 1)$, (ii) $V(x) = (x^2 - 1)^2$.

3. Consider a particle of mass m moving in a one-dimensional potential $V(x)$. Let $x = x_0$ be an unstable fixed point. Define $\eta = x - x_0$ and let
$$\omega = \sqrt{-\tfrac{1}{m}\ \tfrac{d^2V(x)}{dx^2}\Big|_{x=x_0}}\ .$$
Assume that η is small enough so that the quadratic truncation of the potential in the neighbourhood of x_0 is good enough. Show that the most general solution to Newton's equations in this approximation is
$$\eta(t) = A\ e^{\omega t} + B\ e^{-\omega t}\ .$$
Determine the constants A and B for the case when the energy is $E = V(x_0)$ and initial condition at $t = 0$ given by $(\eta = \eta_0, p = p_0)$ with $\eta_0 > 0$ and $p_0 < 0$.

4. Consider the simple pendulum of length L and mass m moving under the influence of gravity.

   (a) Show that the trajectory of the pendulum with energy $E = 2mgL$ given that $\theta = 0$ and $\dot{\theta} > 0$ at $t = 0$ is given by
   $$\theta(t) = \pi - 4\tan^{-1} e^{-\omega t}\quad .$$

   (b) Show that the time-period of the simple pendulum for oscillatory motion can be written as the following integral: $(T = 2\pi\sqrt{L/g})$
   $$\frac{T}{T_0} = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \frac{dy}{\sqrt{1 - K^2\sin^2 y}}\ ,$$
   where θ_0 is the amplitude of oscillations and $K = \sin(\theta_0/2)$.

   (c) By Taylor expanding the denominator of the integrand in part (a) above, show that
   $$\frac{T}{T_0} = 1 + \frac{1}{16}\,\theta_0^2 + O(\theta_0^4)\ .$$

5. Consider a particle of mass m moving in two-dimensions under the action of the quadratic potential (A, B and C are real constants)
$$V(x, y) = A\,x^2 + B\,xy + C\,y^2\ .$$
Analyse and classify the fixed points and their stability for all possible values of the constants.