DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5030 Classical Mechanics **Problem Set 3** **20.8.2021**

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1. Consider the following functionals and obtain the equation(s) of their extremum.
   (a) $S[q] = \int_0^T L(q, \dot{q})$.
   (b) $F[q] = \int_0^T dt \int_0^T dt' q(t)^m q(t')^n$.
   (c) $F[\mathbf{m}, \lambda] = \int_0^T dt \Big(\dot{\mathbf{m}} \cdot \dot{\mathbf{m}} + \lambda \, (\mathbf{m} \cdot \mathbf{m} - 1) \Big)$ with $\mathbf{m} = (m_1, m_2, m_3)^T$.

2. Obtain expressions for the kinetic energy for a particle of mass m in (i) cylindrical polar coordinates (ρ, φ, z), (ii) spherical polar coordinates (r, θ, φ) and (iii) the *parabolic cylindrical* coordinates (σ, τ, z).
   Note: The parabolic cylindrical coordinate system is defined by

$$x = \sigma\tau \ ,$$
$$y = \tfrac{1}{2}(\tau^2 - \sigma^2) \ ,$$

   with the ranges: $-\infty < \sigma < \infty \ , \quad 0 \leq \tau < \infty \ , -\infty < z < \infty$.

3. A (large) rectangular slab of width L lies with its two faces on the planes $z = 0$ and $z = L$. The refractive index of the slab is variable and given by a function $n(z)$. Using Fermat's principle, obtain the equation of the trajectory of a light ray (with angle of incidence θ_i) passing through the slab.

4. [from Marion-Thornton] Consider the surface generated by revolving a line connecting two points (x_1, y_1) and (x_2, y_2) about an axis coplanar with the two points. Find the equation of the curve connecting the two points such that the surface area generated by the revolution is a minimum. Obtain the solution to the equation.

5. Obtain the curve lying on the surface $z = x^3$ connecting the points $(0, 0, 0)$ and $(1, 1, 1)$ that has the shortest arc length('a geodesic'). Use a computer to produce a plot of the surface with the geodesic drawn in the same plot.