DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5030 Classical Mechanics **Problem Set 9** **8.10.2021**

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1. (a) Obtain the explicit expression for an arbitrary rotation matrix in terms of the Euler angles. Let this rotation matrix relate the lab frame to the body frame. By taking the time-derivative, obtain an expression for the angular velocity $\boldsymbol{\omega}$ in terms of the Euler angles and their time derivatives.

   (b) Let R denote an arbitrary time-dependent rotation (i.e., special orthogonal) matrix. Show that the matrix $R^{-1}\cdot dR/dt$ is anti-symmetric.

2. Compute centre of mass, the moment of inertia tensor and the principal axes for the following objects.
   (a) A solid cuboid of uniform mass density ρ_0 and sides L_1, L_2 and L_3.

   (b) A hollow cuboid of uniform surface mass density σ_0 and sides L_1, L_2 and L_3.

   (c) A solid cone of height h and base of radius R is made of a material of uniform mass density ρ_0.

   (d) A hollow cone of height h and base of radius R is made of a material of uniform surface mass density σ_0.

3. A uniform rectangular plate has sides of length a and $2a$. It is free to rotate about one of its corners, which is held fixed. Calculate the moment of inertia tensor about this corner. The plate rotates with angular velocity ω about the diagonal through this corner.
   (a) Find the corresponding (vector) angular momentum of the plate.

   (b) Find the rotational kinetic energy of the plate.

   (c) What torque must be exerted about the corner of the plate in order to maintain this rotation?

4. A cylinder has uniform density ρ_0, radius R and height h. Without doing any calculations and using arguments based on symmetry, draw a diagram showing a set of principal axes for rotations of the cylinder about its centre of mass. Find the principal moments of inertia of the cylinder about its centre of mass. The cylinder rotates about its centre of mass with angular velocity $\boldsymbol{\omega}$. Illustrate the relative directions of the angular momentum $\mathbf{L}$ and the angular velocity $\boldsymbol{\omega}$ for cylinders with (a) $h < \sqrt{3}R$, (b) $h = \sqrt{3}R$, and (c) $h > \sqrt{3}R$.