DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5100 Quantum Mechanics I **Problem Set 10** **5.10.2019**

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1. Obtain the probability density that an electron in the ground state of the hydrogen atom is found at a distance between r and $r+\Delta r$. (Note that you will have to take into account the nontrivial integration measure in spherical polar coordinates.) Show that this probability density has a maximum at the Bohr radius. What is the probability of finding the electron at a radius greater than the Bohr radius?

2. Show that the uncertainty relation for Δx and Δp_x is obeyed by an electron in the ground state of the hydrogen atom.

3. Repeat problem 1 and 2 for an electron in the $2s$ and $2p$ states of the hydrogen atom.

4. Consider an electron in the energy eigenstate $|n,\ell,m\rangle$ of the hydrogen atom. In this state, show that

$$\langle T\rangle_{(n,\ell,m)} = -\frac{1}{2}\langle V\rangle_{(n,\ell,m)} \quad ,$$

where T and V are respectively the kinetic and potential energy of the electron. This is the quantum version of the *classical virial theorem*, which states that if the potential $V \sim r^k$, then the averages $\overline{T}$ and $\overline{V}$ are related by $\overline{T} = k\overline{V}/2$. Hence, show that ($a_0$ is the Bohr radius)

$$\left\langle \frac{1}{r} \right\rangle_{(n,\ell,m)} = \frac{1}{a_0 n^2} \quad .$$

5. The Hamiltonian of the hydrogen atom is, in suitable units,

$$H = \frac{\mathbf{p}^2}{2m} - \frac{e^2}{r} \quad .$$

The quantum mechanical version of the Runge-Lenz vector $\mathbf{A}$ is given by

$$\mathbf{A} = \frac{1}{2m}(\mathbf{p}\times\mathbf{L} - \mathbf{L}\times\mathbf{p}) - e^2\frac{\mathbf{r}}{r} \quad ,$$

where the ordering ambiguity in the first term has been taken care of by appropriate (anti)symmetrisation. Show that $[H,\mathbf{A}] = 0$.