DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5100 Quantum Mechanics I **Problem Set 13** **30.10.2019**

In this assignment, we will carry out the calculations indicated in class during the lectures on time-independent perturbation theory.

1. These are computations related to the Stark effect.

   (a) Let $H_1 = -eEz$. Show that $[H_1, L_z] = 0$.

   (b) Let us order four (ignoring spin) $n = 2$ states in the Hydrogen atom as follows: (We are using the notation $|n, \ell, m\rangle$ to denote the states)

   $$\Big(|2,0,0\rangle,\ |2,1,0\rangle,\ |2,1,-1\rangle,\ |2,1,1\rangle\Big)\ .$$

   In this ordering, show that the matrix of $H_1 = -eEz$ in this four-dimensional subspace is of the form:

   $$\begin{pmatrix} 0 & \langle 2,0,0|H_1|2,1,0\rangle & 0 & 0 \\ \langle 2,1,0|H_1|2,0,0\rangle & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

   where $\langle 2,1,0|H_1|2,0,0\rangle = 3eEa_0$.

   (c) Show that $H + 1$ is diagonal (in this sub-space) in the following basis.

   $$\Big(\tfrac{1}{\sqrt{2}}(|2,0,0\rangle + |2,1,0\rangle),\ \tfrac{1}{\sqrt{2}}(|2,0,0\rangle - |2,1,0\rangle),\ |2,1,-1\rangle,\ |2,1,1\rangle\Big)\ .$$

   (d) Hence draw the energy splitting diagram in this case.

2. These are computations related to the Zeeman effect.

   (a) Let $H_1 = -\frac{eB}{2m_e c}(L_z + 2S_z)$. Verify that $[H_1, L_z] = 0$ and $[H_1, S_z] = 0$.

   (b) Let us order eight (including spin) $n = 2$ states in the Hydrogen atom as follows: (We are using the notation $|n, \ell, m, m_s\rangle$ to denote the states)

   $$\Big(|2,0,0,\tfrac{1}{2}\rangle,\ |2,1,0,\tfrac{1}{2}\rangle,\ |2,1,-1,\tfrac{1}{2}\rangle,\ |2,1,1,\tfrac{1}{2}\rangle,$$
   $$|2,0,0,-\tfrac{1}{2}\rangle,\ |2,1,0,-\tfrac{1}{2}\rangle,\ |2,1,-1,-\tfrac{1}{2}\rangle,\ |2,1,1,-\tfrac{1}{2}\rangle\Big)\ .$$

   In this basis, show (using part (a)) that the H_1 is diagonal and compute all diagonal elements.

   (c) Hence draw the energy splitting diagram in this case.